Optimizing Platform Survivability Using Scenario Analysis and Stochastic Linear Programs

A Research Proposal

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Organization

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- **Research Proposal**
- **The Next Steps**
- **Tool Application**
- **Summary**
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Introduction

- **The Army in Transformation**
  - **A new “objective force” is being created.**
- **An Objective Force with Design Goals of:**
  - **Increased responsivity and deployablity,**
  - **Increased agility and versatility,**
  - **Increased lethality and survivability,**
  - **Decreased logistics footprint.**
- **Obstacles in Meeting these Goals include:**
  - **Increased threat weapons lethality,**
  - **A diverse threat,**
  - **Different operational situations, and**
  - **Varied mission requirements and terrain types.**

Some Definitions

- **A platform is:**
  - **A single instance of a vehicle such as an Abrams tank, a Bradley personnel carrier or other vehicle.**
  - **The term platform is used to establish a reference for illustrating the application of stochastic linear programming and scenario analysis.**
- **Survivability is the end result of:**
  - **Making it through an encounter with a threat system with some level of effectiveness or functionality intact.**
  - **This definition does not assume that a platform will make it through unscathed simply that it will not be destroyed.**

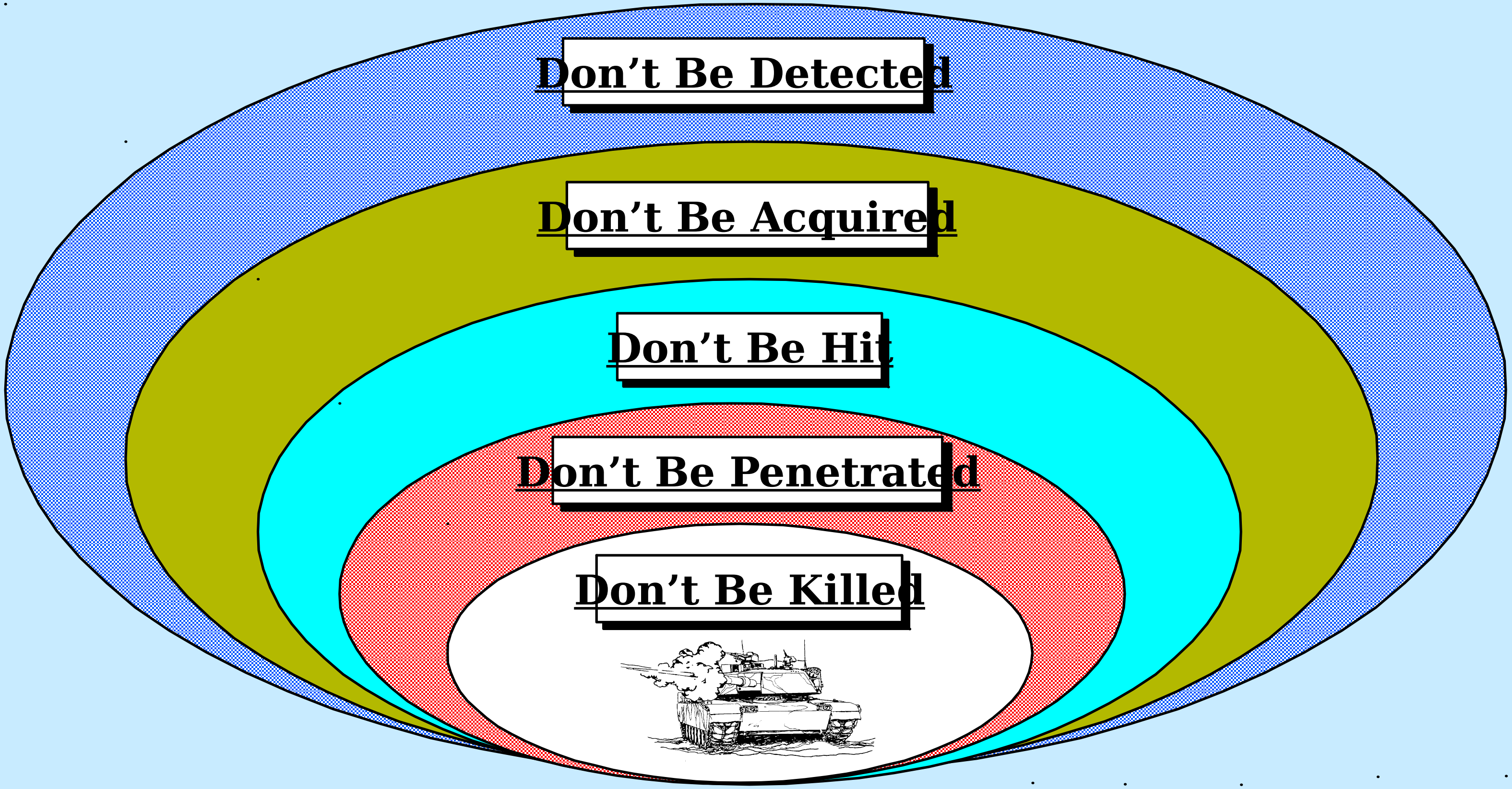
Examples of Design Trade Offs

- Increase deployability implies decreasing platform burdens such as:
  - Weight (which includes armor) and electrical power.
  - Volume and dimension.
- Increased survivability implies
  - Maintaining battlefield effectiveness
  - Meeting or exceeding survivability of today's platforms
  - Reducing platform burdens
- A budget that is 60% of what it was 10 years ago.

Motivating Question

- **Question: "How can one maintain platform effectiveness while addressing the design issues of deployability, responsivity, agility, versatility, survivability, lethality and sustainability in addition to meeting the increased survivability goal?"**

Community Approach



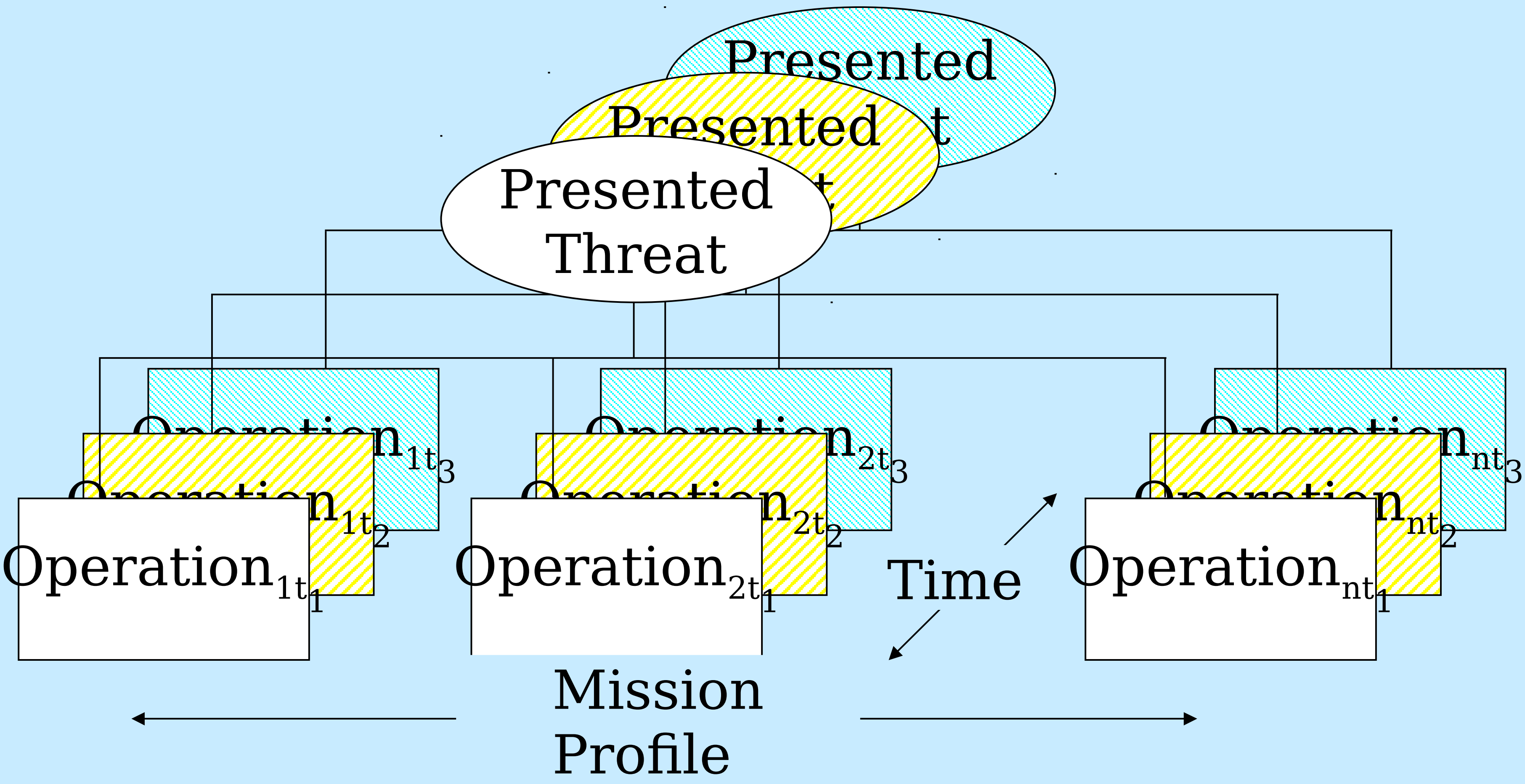
Observations

- **Platform design considers:**
  - **Many diverse missions and tasks to perform.**
  - **Many design constraints, not just survivability.**
  - **A survivability design task that must consider a changing threat.**
- **The survivability suite:**
  - **The collections of survivability options is finite.**
  - **There are MANY design approaches and technology solutions for addressing the areas of platform survivability.**
  - **There may be performance synergies (positive or negative) between various survivability options.**
- **The threat:**
  - **The threat space may be large, but it is finite.**
  - **The threat varies with mission, location and environment.**
  - **The threat also varies with time (the threat evolves and responds).**

Research Question

- How can a designer field a platform that
  - Considers the diverse mission requirements,
  - Maintains effectiveness,
  - Meets the design goals,
  - Survives on future battlefields, and
  - Lives within budgetary concerns?
- The answer is a combination of scenario analysis and stochastic programming approaches
  - Scenario analysis captures variations in time and mission.
  - Stochastic programming captures uncertainty within a given scenario.

Scenario Analysis



Stochastic Programming

- **Chance constrained linear programming**
  - **“Here and now” approach.**
  - **Finds a solution that meets the constraints for all but a small fraction of the threat array.**
  - **This topic will be carried forward in this discussion.**
- **Stochastic programming with recourse**
  - **“Wait and see” approach.**
  - **Is representative of the design process when the designer considers what approaches to “design in” versus what “technology solutions” are added later.**
  - **This topic is included in the proposal but not in this presentation.**

Research Statement

- **As English: how can I field a platform that**
  - **Considers the scenario specific mission requirements,**
  - **Meets the design goals,**
  - **Considers the budgetary concerns, and**
  - **Survives on future battlefields**
- **As an optimization problem:**
    - » **min $z = w^T x$**
  - **subject to**
    - » **Ax □ b □ known requirements (volume, power, cost, etc.)**
  - **and**
    - » **$P_s(S_s, □(□))$ □ □ □ survivability constraint**

Chance Program

- **As English: how can I field a platform that**
  - **Considers the scenario specific mission requirements,**
  - **Meets the design goals,**
  - **Considers the budgetary concerns, and**
  - **Survives engagements with all but a small fraction of the threats**
- **The problem as a chance constrained program is:**
  - » **min** $z = w^T x$
  - **subject to**
    - » $Ax \le b$

$$x \in X = \{0,1\}^{n_X} \quad \text{where } n_X = \#(S_m)$$

  - **and**
    - » **P[P**$_s(S_s, \cdot(\cdot))$ **] **

$$S_s = \{ s_j \in S_m \mid x_j = 1, j = 1, \ldots, n_X \}$$

Definitions

- A_D design approaches for avoiding detection,
- A_H technology solutions for avoiding hit,
- A_P design approaches for avoiding penetration,
- A_K design approaches to avoid being killed,
- S_m a set of survivability measures
  - □ A_D □ A_H □ A_P□□ A_K
- S_s a survivability suite under consideration
  - □ S_s □ S_m
- Note: all sets are finite sets.

Probability of Survival

- Probability of survival is defined as follows:
  - $$P_s\left(S_s, \mathbf{x}(\omega)\right) = \prod_{i=1}^{4}\left[1 - P_i\left(S_s, \mathbf{x}(\omega)\right)\right]$$
- Where $P_i(S_s, \mathbf{x}(\omega))$ is defined as the probability that the threat
  - $P_1(S_s, \mathbf{x}(\omega))$: detects the platform,
  - $P_2(S_s, \mathbf{x}(\omega))$: hits given platform detection,
  - $P_3(S_s, \mathbf{x}(\omega))$: penetrates the platform given hit,
  - $P_4(S_s, \mathbf{x}(\omega))$: kills the platform given hit,

  With survivability suite S_s applied to the platform

Solution Approach

- **This approach is intuitive:**
  - **It frames the problem as a typical engineering design problem, with**
  - **Costs and burdens as design constraints, and**
  - **The probability of survival treated as a design goal to be met at some desired threshold.**
- **This approach uses:**
  - **Burdens as a series of deterministic design constraints that restrict the choice of survivability measures, and**
  - **Probability of survival as another constraint which we will meet at a specified level of tolerance.**

Simple Example

- **For an unrealistic example for illustration, assume:**
  - **Only one survivability measure per avoidance area,**
  - **No interaction among survivability measures, nor**
  - **Interaction among avoidance areas.**
- **Define:**

$$T_j(\mathbf{w}) = \left\{ \begin{array}{ll} \ln\left[1 - \mathrm{P}_1\left(s_j, \mathbf{x}(\mathbf{w})\right)\right] & \forall s_j \in \left(S_S \cap A_D\right) \\ \ln\left[1 - \mathrm{P}_2\left(s_j, \mathbf{x}(\mathbf{w})\right)\right] & \forall s_j \in \left(S_S \cap A_H\right) \\ \ln\left[1 - \mathrm{P}_3\left(s_j, \mathbf{x}(\mathbf{w})\right)\right] & \forall s_j \in \left(S_S \cap A_P\right) \\ \ln\left[1 - \mathrm{P}_4\left(s_j, \mathbf{x}(\mathbf{w})\right)\right] & \forall s_j \in \left(S_S \cap A_K\right) \end{array} \right\}$$

Solution Requirements

The chance constraint is convex under the following general conditions:

- **Let ω have a finite discrete distribution described by:**

$$P(\omega = \omega_j) = p_j,\ j = 1,\ldots,\upsilon \text{ and } p_j > 0\ \forall j.$$

then for

$$\alpha > 1 - \min_{j \in \{1,\ldots,\upsilon\}} p_j$$

the feasible set defined by:

$$B(\alpha) = \left\{ x \mid P\left[\omega \mid T(\omega)x \geq h(\omega)\right] \geq \alpha \right\}$$

is convex where $h(\omega)=\ln(\omega)$.

The Next Steps

- **Further develop the functional form of $P_s(S_s, \square(\square))$**
  - **This formulation does not include factors such as threat recognition and classification. This is assumed for this proposal.**
- **Questions to be answered**
  - **Are there reasonable approximations which can be used to model $P_s(S_s, \square(\square))$, and if so, under what conditions are they valid?**
- **Evaluate solution approaches for $P_s(S_s, \square(\square))$**
  - **It does not appear necessary for the form of $P_s(S_s, \square(\square))$ to be completely known to evaluate solution approaches.**

Analytical Approach

- **Using recourse programming to reduce the dimensionality of the set of survivability measures.**
  - **With recourse programming, if you meet the constraints, you have a potential solution. So, make a loose set of requirements, and use them to screen the candidates.**
- **Use the designed in survivability measures to find a solution. Then the recourse becomes the technology solution necessary to reach the true design goal.**

Analytical Approach

- **Use a chance programming approach to drive to a true optimal solution.**
  - **Chance programming accepts that you will not be able to address all the threats, use this feature to find a satisfactory solution for most all cases.**
- **Use a different form of the objective to create weighted solutions.**
  - **Objective weights can be used to give preference to existing or status quo approaches versus notional approaches by giving higher weights to the notional approach.**

Summary

- **The survivability of both the current and future fleets of vehicles will rely on "survivability suites" for protection, not just an armor solution.**
- **Optimizing platform survivability can be approached using stochastic linear programming and scenario analysis.**
- **There are solution approaches available to solve the chance program in various forms.**
  - **The approaches depend on the information available regarding the underlying distribution of $P_s(S_s, \square(\square))$, and**
  - **The approaches may depend the definition of $P_s(S_s, \square(\square))$.**

Conclusions

- **The Army needs a selection tool for determining an optimal survivability suite**
  - **Considering the threat space,**
  - **The design goals, and**
  - **The available design approaches.**
- **Stochastic programming and scenario analysis provides a rigorous methodology for determining a “best” suite of survivability measures for a given platform.**
- **Chance programming may also provide a way to trade off various design criteria in meeting single engagement and force survival requirements.**

Discussion

Are there any questions?

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